MERITS OF MINIMIZING THE SQUARES OF THE DEFLECTION COMPONENTS VERSUS MINIMIZING THE SQUARES OF GEOIDAL UNDULATIONS FOR DETERMINING BEST FITTING DATUMS

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من المعلوم أن أحسن سطح اسناد محلى يمكن استنباطه يجعل مجموع مربعات أبتعاد الجيويد عن الالبسيويد أقل مايمكن أو جعل مجموع مربعات الانحراف على الرأسى أقل مايمكن .. وقد تم فى هذا البحث استخدام الشرط الثانى للحصول على أحسن سطح أسناد لجمهورية مصر العربية .

وقد أثبت البحث أن استخدام أى شرط من الشرطين قد يؤدى الى تقليل قيمة الحيود عن الرأسى أو قيم ابتعاد الجيويد عن الالبسيويد .. ولكن أحسن القيم يمكن الحصول عليها عندما يتحقق الشرطان معا .. وقد تم هذا باستخدام الشرط الثانى مع تكرار الحلول وباختيار قيم مختلفة لابتعاد الجيويد عن الالبسيويد عند نقطة البداية .

ABSTRACT

In the past , the Egyptian triangulation network was calculated on HELMERT-1906 and HYFORD-1910 ellipsoids and the geodetic maps were produced on HELMERT-1906 using transverse Mercator projection . Reducing the observations to the reference ellipsoid was ignored due to the lack of data about the geoid in Egypt during that period. Neglecting these reductions causes distortions in the scale and the orientation of the network . Now , data about the geoid in Egypt are available and it was found reasonable to test the reliability of applying the condition of minimization ($\Sigma\ \theta^2 = min$) ,specially after the other condition of minimization ($\Sigma\ N^2 = min$) was previously tested ,to achieve a best fitting ellipsoid for Egypt.

I- Introduction

Geodetic observations are taken relative to the geoid which is an irregular surface and difficult to be represented mathematically . Accordingly the geoid is replaced by an ellipsoid which is the nearest regular mathematical figure to that geoid . This ellipsoid is used for the computations and the projection process . Consequently it is necessary to reduce the observations from the geoid to that ellipsoid . The errors arising from neglecting this reduction have a great effect on the accuracy of the coordinates *(Bomford 71)*. In order to minimize these errors , the geodetic datum should be well defined .

The geodetic datum is comprised of an ellipsoid of revolution with specified size and shape , fixed in position w.r.t. the geoid through specified parameters . An ellipsoid approximating the shape of the global geoid and having geometrical center at the earth's center of gravity forms a (global datum)or a (geocentric datum) or a (world wide datum) , it has the advantage that all the geodetic coordinates will refer to a unique datum allover the world. Geocentric datums have a disadvantage because at some areas the deviation between the selected ellipsoid and the geoid are large.This has its effect on the process of reducing the observations, in which all theinvolved corrections must be considered . On the other hand the ellipsoid approximating the geoid in a limited region and having a specified relationship with the geoid at the origin (initial point) of the network, forms a regional or local datum . The local datum is usually refered to as a best fitting datum . The geometric center of the best fitting ellipsoid doesn't generally coincides with the center of mass of the earth . Best fitting datums have the advantage of minimizing the deviations from the geoid , this makes the corrections applied for the reduction of observations relatively small and thus may be neglected in some cases (*M. Nassar 87*). Local datums may be applied for a small areas only and may not be suitable for other distant areas.This leads us to the use of different reference ellipsoids for different areas which results in a large number of datums to be excisted throughout the world (*H. Moritz 81*)

2- MATHIMATICAL MODEL

The best fitting ellipsoid an be achieved under one of the following conditions (*Heiskanen & Moritz 67*):

$$\sum_{k=1}^{n} N_k^2 = \min. \tag{1}$$

or

$$\sum_{k=1}^{n} (\xi_k^2 + \eta_k^2) = \min. \tag{2}$$

where

N geoidal undulation

ξ η deflection components.

n total number of stations under considerations

In this study only the second condition will be considerd as the first one has been previously examined (*Alnagar, D. & Shaker, A 1988*).

2-1 LEAST SQUARES SOLUTION OF THE SECOND CONDITION

Vening Meinesz formulae for the change in the deflection of the vertical components (ξ , η) as a function of changing the geodetic datum are given by :

$$\begin{aligned} \delta\xi = {} & (\cos\phi_i \cos\phi + \sin\phi_i \sin\phi \cos\Delta\lambda)\, \delta\xi_i - \sin\phi \sin\Delta\lambda\, \delta\eta_i \\ & - (\sin\phi_i \cos\phi - \cos\phi_i \sin\phi \cos\Delta\lambda)\, (\delta N_i/a + \delta a/a + \sin^2\phi_i\, \delta f) \\ & - 2 \cos\phi\, (\sin\phi - \sin\phi_i)\, \delta f \end{aligned} \tag{3}$$

$$\begin{aligned} \delta\eta = {} & \sin\phi_i \sin\Delta\lambda\, \delta\xi_i + \cos\Delta\lambda\, \delta\eta_i + \cos\phi \sin\Delta\lambda\, (\delta N_i/a \\ & + \delta a/a + \sin^2\phi_i\, \delta f) \end{aligned} \tag{4}$$

for (n) points the parametric least squares will take

the form :

$$
{}_{2n}V_{1} = {}_{2n}A_{5} \cdot {}_{5}X_{1} + {}_{2n}L_{1}
$$

$$
\begin{vmatrix} \xi'_1 \\ \xi'_2 \\ \cdot \\ \xi'_n \\ \eta'_1 \\ \xi'_2 \\ \cdot \\ \eta'_n \end{vmatrix}
=
\begin{vmatrix}
{}_1a_1 & {}_1a_2 & {}_1a_3 & {}_1a_4 & {}_1a_5 \\
{}_2a_1 & {}_2a_2 & {}_2a_3 & {}_2a_4 & {}_2a_5 \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
{}_{2n}a_1 & {}_{2n}a_2 & {}_{2n}a_3 & {}_{2n}a_4 & {}_{2n}a_5
\end{vmatrix}
\cdot
\begin{vmatrix} \delta\xi_i \\ \delta\eta_i \\ \delta N_i \\ \delta a \\ \delta f \end{vmatrix}
+
\begin{vmatrix} \xi_1 \\ \xi_2 \\ \cdot \\ \xi_n \\ \eta_1 \\ \eta_2 \\ \cdot \\ \eta_n \end{vmatrix}
\qquad (5)
$$

Where

V is the vector of the deflection component after solution .

A is the coefficient matrix .

X corrections applied to the approximate parameters given at the initial point.

L Vector of the deflection components befor solution .

n is the number of data points .

The solution of this system is as follows

$$
X = -(A^{T} \; A)^{-1} \; A^{T} \; L \qquad (6)
$$

and by back substitution in

$$
V = A \; X + L
$$

the new deflections on the new datum are calculated.

It should be mentioned that the coefficients of δN_i and δa in equations (3 & 4) are equal, which produces a singularity in the solution for the five parameters . Accordingly , when solving for the second condition , certain constraint should be applied during the solution as will be explained later. Also the above mathematical model will be tested here using the available data in Egypt.

3- DATA UNDER CONSIDERATIONS

A grid of geoidal undulation for Egypt every 15' latitude and 15' longitude from 22° N to 32° N and from 25°E to 36° E are available (*Alnagar 86*). These undulations were given on WGS-72.

- Calculation of ξ and η from goidal undulation N

The calculation of ξ, η from the goidal undulations N, on WGS-72 ,are based on the idea of the geoidal slope,fig(1),as follows.

first: for points in the North-south direction

$$\xi_i = \left(\mathrm{Tan}^{-1} \frac{N(i)-N(i+1)}{\mathrm{dist.}(i,i+1)} + \mathrm{Tan}^{-1} \frac{N(i-1)-N(i)}{\mathrm{dist.}(i-1,i)} \right) /2 \qquad (7)$$

$$\xi_i = (\xi_1 + \xi_2) /2$$

second: for points in the East-West direction

$$\eta_j = \left(\mathrm{Tan}^{-1} \frac{N(j)-N(j+1)}{\mathrm{dist.}(j,j+1)} + \mathrm{Tan}^{-2} \frac{N(j-1)-N(j)}{\mathrm{dist.}(j-1,j)} \right) /2 \qquad (8)$$

$$\eta_j = (\eta_1 + \eta_2) /2$$

- Transformation of ξ , η and N from WGS 72 to Helmert 1906

This transformation was made to enable us to calculate a best fitting ellipsoid from Helmert-1906 data and from WGS-72 data to test the validity of the results . The transformation process was done by using Vening Mienesz formulae (3 & 4)

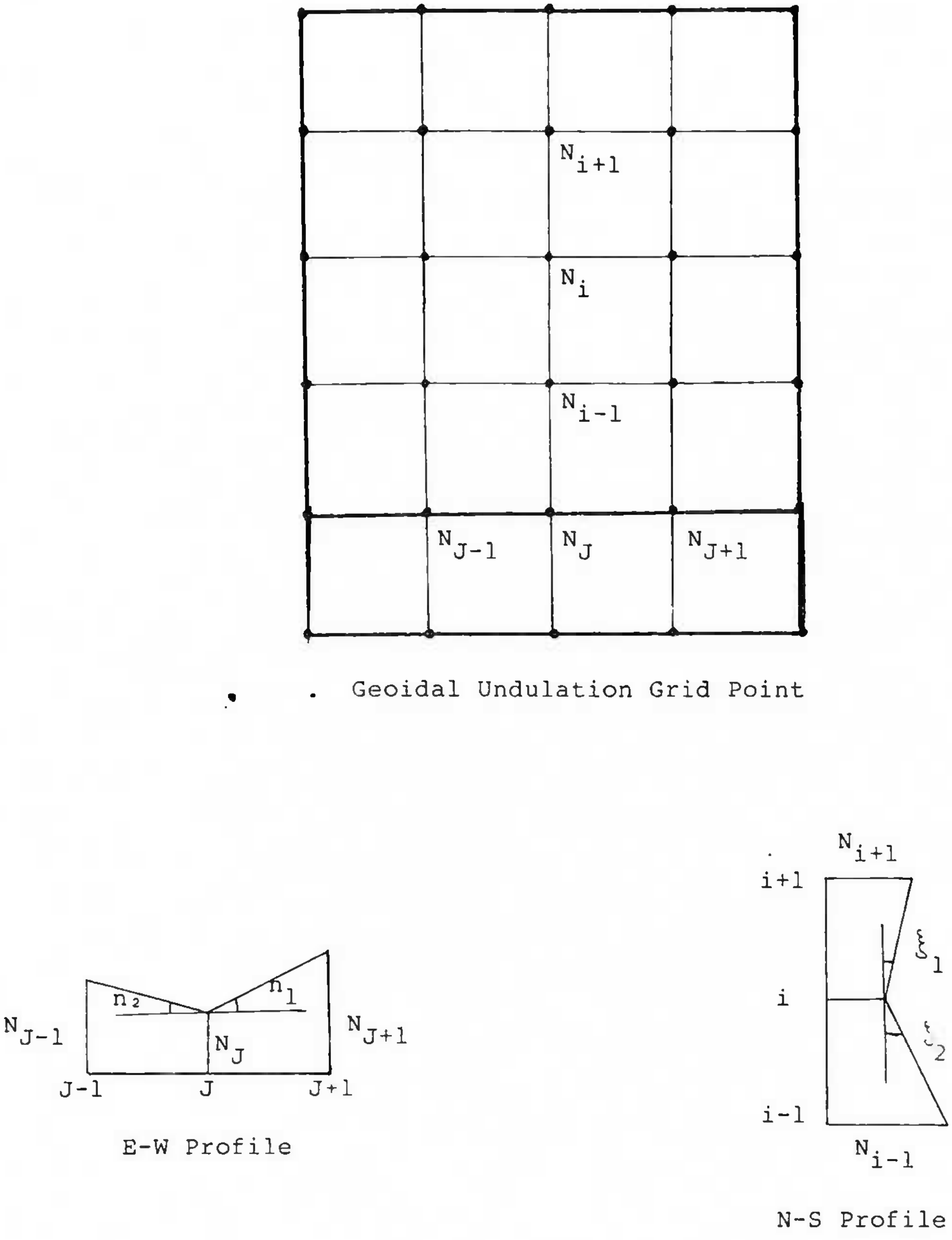

N i+1
N i
N i-1
N J-1
N J
N J+1
. Geoidal Undulation Grid Point
η2
η1
N J-1
N J
N J+1
J-1
J
J+1
E-W Profile
N i+1
i+1
ξ1
i
ξ2
i-1
N i-1
N-S Profile

Fig. (1)

4 DIFFERENT SOLUTIONS APPLIED FOR THE BEST FITTING ELLIPSOID

Different solutions were carried out by varying the parameters to study the effect of these parameters on the results and to obtain the best among them . These solutions are ;

1-a Using Helmert 1906 data and constraining the value of N_i at the initial point to be 0.0 mt.*(Helmert datum data)*

1-b Using WGS-72 data and constraining the value of N_i at the initial point to be 13.47 mt.*(WGS datum data)*

1-c Using WGS-72 data and constraining the value of N_i at the initial point to be 0.0 mt.

1-d Using WGS-72 data and constraining the value of N_i to be 0.863 mt

1-e Using WGS-72 data and constraining the value of N_i to be 1.0 mt.

2-a Using Helmert 1906 data and constraining N_i & f to 0.0 mt and 1/298.

2-b Using WGS-72 data and constraining N_i & f to 13.478 mt and 1/298.26

2-c Using WGS-72 data and constraining N_i & f to 0.85 mt. and 1/298.26

2-d Using WGS-72 data and constraining N_i & f to 1.0 mt and 1/298.26

3-a Using Helmert 1906 data and constraining (N_i , a , f) to 0.0 mt. , 6378200 and 1/298.3

3-b Using WGS-72 data and constraining(N_i, a , f) to 13.478 , 6378135 and 1/298.26.

3-c Using WGS-72 data and constraining (N_i, a, f) to 1.0 ,6378135 and 1/298.26

4-a Using Helmert 1906 data and constraining (ξ_i, η_i, N_i) to 3.93 , 0.0 and 0.0

4-b Using WGS-72 data and constraining (ξ_i, η_i, N_i) to 3.48 , -4.73 and 13.478 .

4-c Using WGS-72 data and constraining (ξ_i, η_i, N_i) to 3.48 , -4.73 and 1.00

5- RESULTS

The results of (ξ_i, η_i, N_i, a & f) obtained from the different solutions are given in table (1), and in order to find out the best solution , the values of the summations $\Sigma\xi^2$, $\Sigma\eta^2$, $\Sigma\theta^2$ and ΣN^2 are calculated for every solution and tabulated with their estimated variances , Table (2).

From the results given in table (1) and table (2) it can be seen that solution (2-d) is the best solution which minimize not only $\Sigma(\xi^2 + \eta^2)$ but also ΣN^2 as can be seen from fig(2),fig(3). Accordingly the parameters produced for the best datume are as follows

a=6377771.6 mt f=1/298.26

$\xi = 3.197''$ $\eta = -4.511''$ N= 1.00 mt.

This datum gave 4.5% minimization for $\Sigma(\xi^2 + \eta^2)$ and 94.45% for ΣN^2 w.r.t WGS 72 .

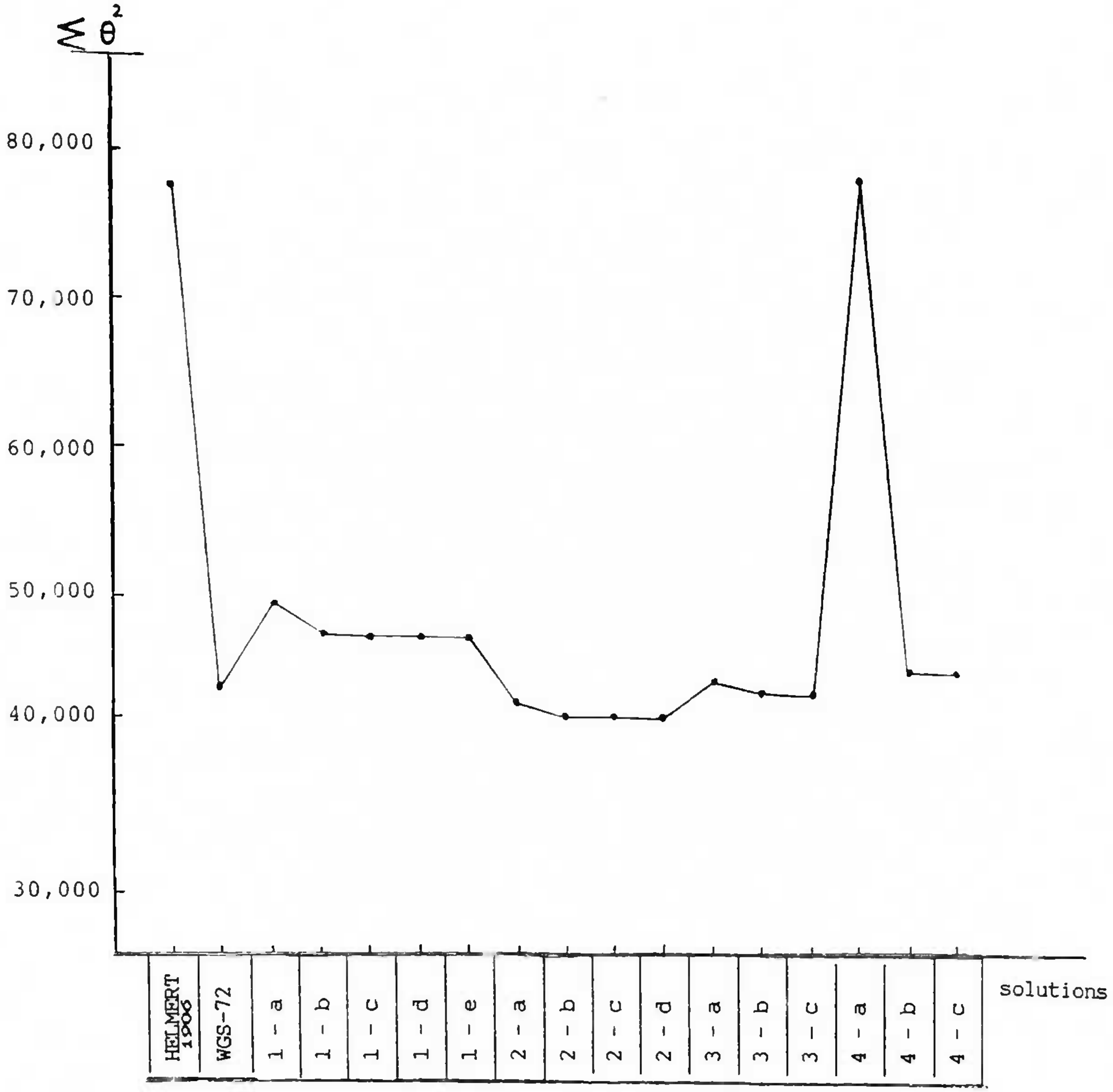
	ξ''_i	η''_i	N_i (m)	a (m)	f
1 - a	3.87 ±0.12	-4.68 ±0.086	0.00 *	6377466.75 ±45.131	1/305.423 ±7.09E-6
1 - b	3.86 ±0.117	-4.67 ±0.084	13.478 *	6377456.91 ±44.107	1/305.383 ±6.93E-6
1 - c	3.86 ±0.117	-4.67 ±0.084	0.00 *	6377470.38 ±44.019	1/305.383 ±6.91E-6
1 - d	3.86 ±0.117	-4.67 ±0.084	0.863 *	6377469.52 ±44.026	1/305.383 ±6.91E-6
1 - e	3.86 ±0.117	-4.67 ±0.084	1.00 *	6377469.38 ±44.019	1/305.383 ±6.92E-6
2 - a	3.10 ±0.095	-4.509 ±0.078	0.00 *	6377812.60 ±33.106	1/297.3 *
2 - b	3.20 ±0.095	-4.51 ±0.078	13.478 *	6377759.22 ±32.827	1/298.26 *
2 - c	3.197 ±0.09	-4.511 ±0.07	0.85 *	6377771.84 ±32.826	1/298.26 *
2 - d	3.197 ±0.095	-4.511 ±0.078	1.00 *	6377771.69 ±32.827	1/298.26 *
3 - a	3.75 ±0.079	-4.359 ±0.079	0.00 *	6378200 *	1/297.30 *
3 - b	3.82 ±0.078	-4.07 ±0.078	13.478 *	6378135 *	1/298.26 *
3 - c	3.807 ±0.78	-4.37 ±0.078	1.00 *	6378135 *	1/298.26 *
4 - a	3.93 *	0.00 *	0.00 *	6378032.50 ±55.843	1/299.786 ±7.67E-6
4 - b	3.48 *	-4.73 *	13.478 *	6377436.10 ±41.872	1/304.392 ±5.75E-6
4 - c	3.48 *	-4.73 *	1.00 *	6377448.54 ±41.872	1/304.392 ±5.75E-6

* Constraint parameters

Table (1)

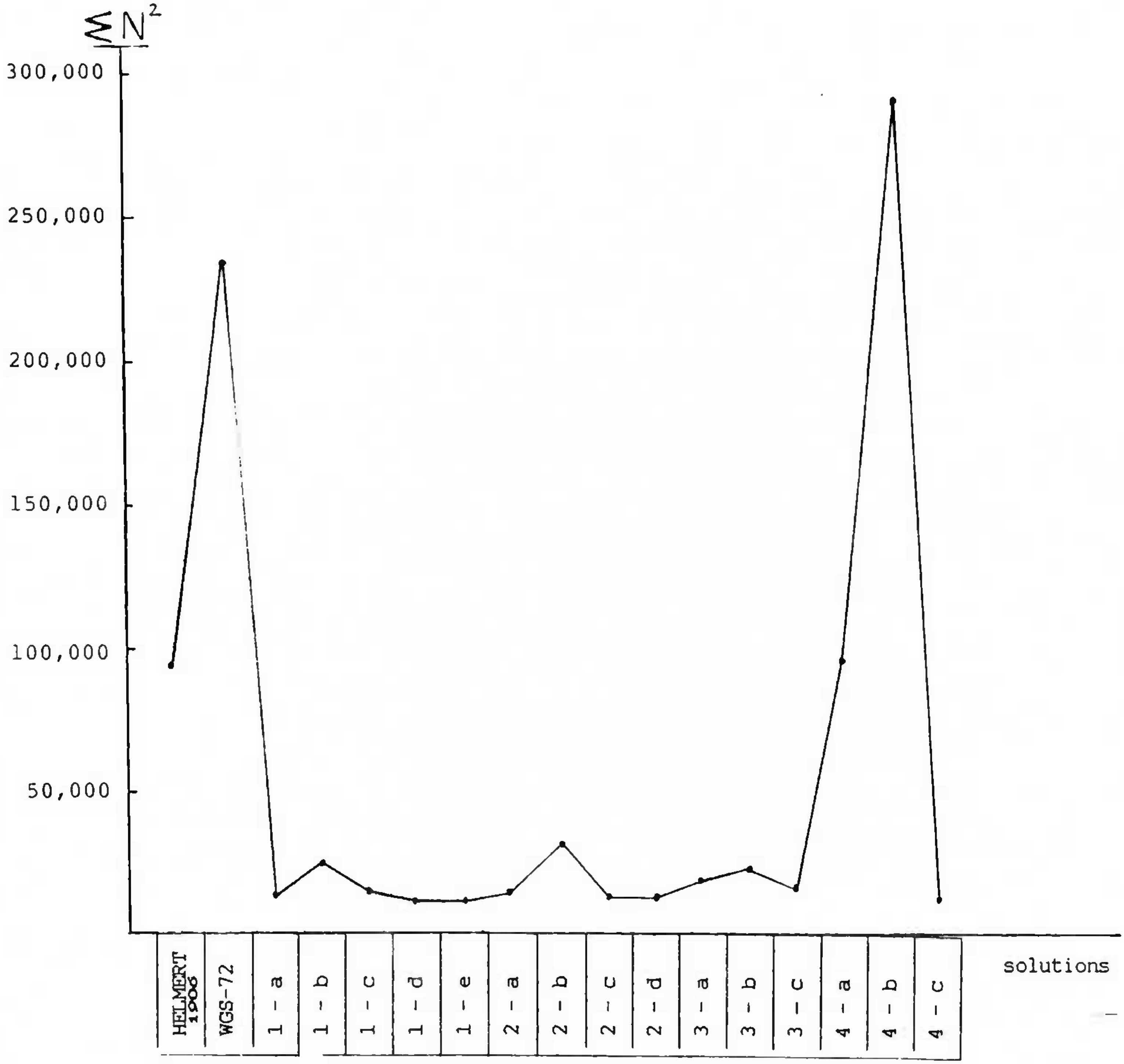
	ΣN^2	$\Sigma \zeta^2$	$\Sigma \eta^2$	$\Sigma \theta^2$	$\hat{\sigma}_0^2$
HELMERT 1906	94231.97	24011.80	53162.50	77174.30	
WGS-72	234156.20	23908.78	17971.46	41880.24	
1 - a	11932.55	32654.51	14997.21	47651.72	1.005
1 - b	278483.5	30515.57	14991.58	45507.15	1.008
1 - c	16232.43	30515.54	14831.17	45346.71	1.004
1 - d	12921.78	30515.56	14831.17	45346.73	1.004
1 - e	12648.99	30515.57	14831.17	45346.74	1.004
2 - a	15296.82	24637.24	16006.65	40643.89	0.900
2 - b	301279.80	24393.08	15598.43	39991.51	0.885
2 - c	13020.11	24393.08	15598.43	39991.51	0.885
2 - d	12990.89	24393.08	15598.43	39991.51	0.885
3 - a	18234.24	23682.09	18471.76	42153.85	0.933
3 - b	238816.2	23658.75	17753.48	41412.23	0.917
3 - c	17445.51	23659.84	17425.32	41085.16	1.782
4 - a	96376.48	24819.41	52954.09	77773.50	1.721
4 - b	290716.4	28885.19	14841.97	43727.16	0.968
4 - c	11944.28	28885.18	14841.95	43727.13	0.967

Table (2)



Effect of Different Solutions on $\Sigma\theta^2$

Fig. (2)



Effect of Different Solutions on ΣN^2

Fig. (3)

6-CONCLUSION AND RECOMMENDATIONS

From all the solutions and their results , it is usfull to state the following :

a- Fixing the orientation parameters, (ξ_i , η_i & N_i) at the initial point, to any value, and leaving only the size and shape of the ellipsoid to change freely is not a convinient choice for achieving a best fitting datum.

b- Fixing (a , f), the size and the shape of the ellipsoid, and changing (ξ_i, η_i, N_i) gives a suitable results .

c- Fixing (f) the flattening of the ellipsoid and changing the other parameters (ξ_i, η_i, N_i & a) produces good results .

d-The assumed values of (N_i) has an effect on the resulted (ξ_i , η_i ,a & f) and the minimized values .

e- To get good results from Vening Miniesz formulae a suitable value for (N_i) should be assumed and this is done by trial with some solutions to obtain the best minimization .

f- The values $\Sigma\xi^2$ and $\Sigma\eta^2$ on WGS-72 before minimization were found to be suitable values when compared with the corresponding values produced after minimizations. i.e. WGS-72 before minimization satisfies the condition $\Sigma\theta^2$= min. within certain limit .

g- The summations of ΣN^2 indicate a large shift between WGS-72 and the geoid in Egypt and that is logic because WGS-72 is a globale system .

h- The value $\Sigma\xi^2$ at Helmert-1906 is very close to the crrosponding values at WGS-72 and also to the new produced datum

i- $\Sigma\eta^2$ at HELMERT-1906 is large compared with the corresponding values at WGS-72 and the new produced datum .

j- The large value $\Sigma\eta^2$ at Helmert-1906 made the total value of $\Sigma\theta^2$ large also if compared with that value at WGS-72 and of the new produced datum .

k- Also ΣN^2 at Helmert-1906 is not suitable if compared with the corresponding value at the new proposed datum .

1-The values of $\Sigma\ N^2$ & $\Sigma\ \theta^2$ produced from satisfying the first condition only *(Alnagar, D. & Shaker, A. 1988)* were 20343.57 and 59029.21 which are relativly large compared with the new produced values , 12990.89 for $\Sigma\ N^2$ and 39991.51 for $\Sigma\ \theta^2$.

From the above results and conclusions , the following are recommended :

- The condition of $\Sigma\ N^2$ = min is not recommended to be used alone for determining best fitting datums. However it is very usefull to be used for determinig a reasonable value of the undulation N_i at the datum initial point, which is very essential as a starting value to be constraint for the solution using the other condition $\Sigma\ \theta^2$=min . The best value for N_i is achieved by trial through the repetition of the second condition.
- When another data are available , another best fitting ellipsoid must be calculated using the improved geoid .
- It is advisable to readjust the Egyptian networks using the new proposed datum taking into consideration the gravimetric effects .

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